

Computer Algebraic Systems (CAS) and Calculators

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Topics

1. What is CAS?
2. Calculators with CAS
3. Some essential CAS functions
4. Education and CAS

What is CAS?

CAS: Computer Algebra Systems

Symbolic manipulation of algebraic objects and interpreting and parsing mathematical expressions

Display mathematical expressions as “it would be written” (pretty print instead of one-line expression)

What is CAS?

Symbolic answers are allowed without the calculator requiring the user to enter numeric values for each variable

Example:

$$x^2 + a * x - b$$

CAS-enabled calculators are the first to simplify most rational expressions exactly

Example:

$$(7 / (6 + \sqrt{7})) \text{ simplifies to } (-7 \sqrt{7} + 42)/29$$

Casio fx-5500 (1986)

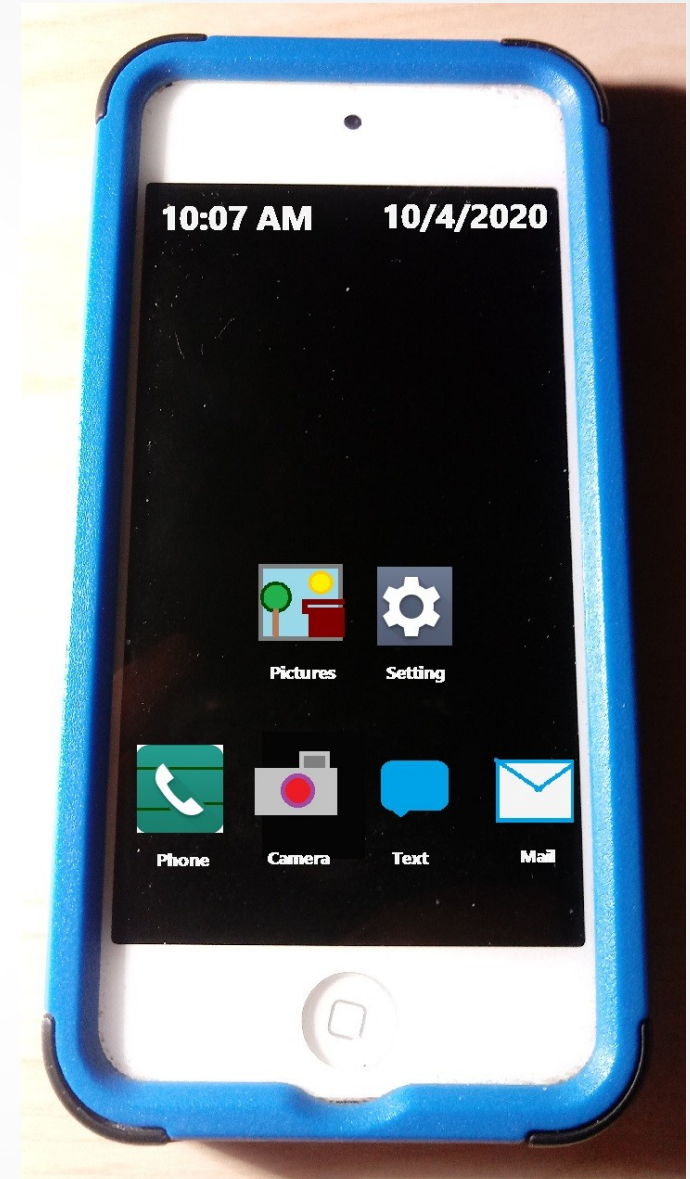
- One of the first, if not the first, calculators with CAS functions:
 - Simplification of Polynomials
 - Polynomial Expansion
 - Polynomial Factorization
 - Range of CAS functions limited to arithmetic operators, powers, and roots
- Landscape Design, QWERTY Keyboard
- Featured on Ledudu's Casio page:
<https://casio.ledudu.com/pockets.asp?type=79&lg=eng>

Calculators That Have CAS

- Hewlett Packard
 - HP 28S/C Series
 - HP 48S/G, 49, 50G Series
 - HP Prime
- Texas Instruments
 - TI-89/92 Series
 - TI nSpire CAS Series
- Casio
 - fx-5500 (NOT 5500L)
 - Algebra FX 2.0
 - Classpad Series

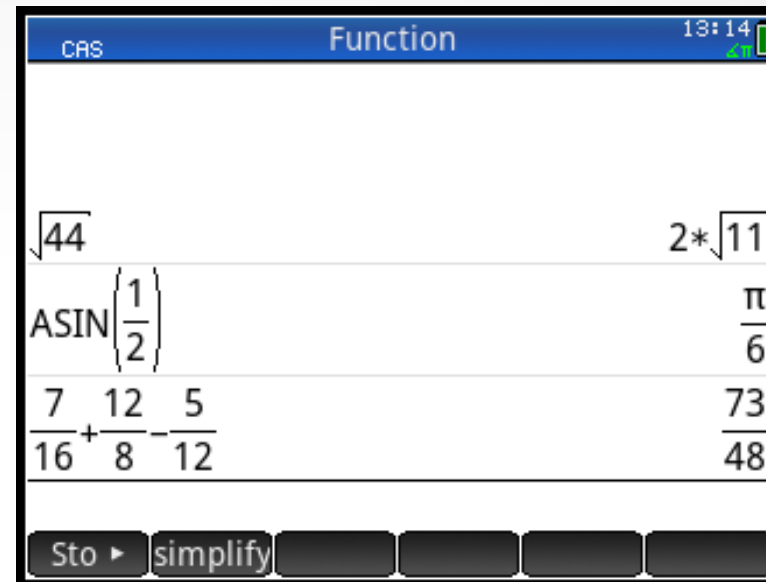
CAS Apps on Your Phone!

- Maxima on Android
- Wolfram Alpha (also on computers)
- MathStudio (iOS)



Some Essential CAS Functions

Simplification: Square
Roots, Terms of Pi,
Fractions

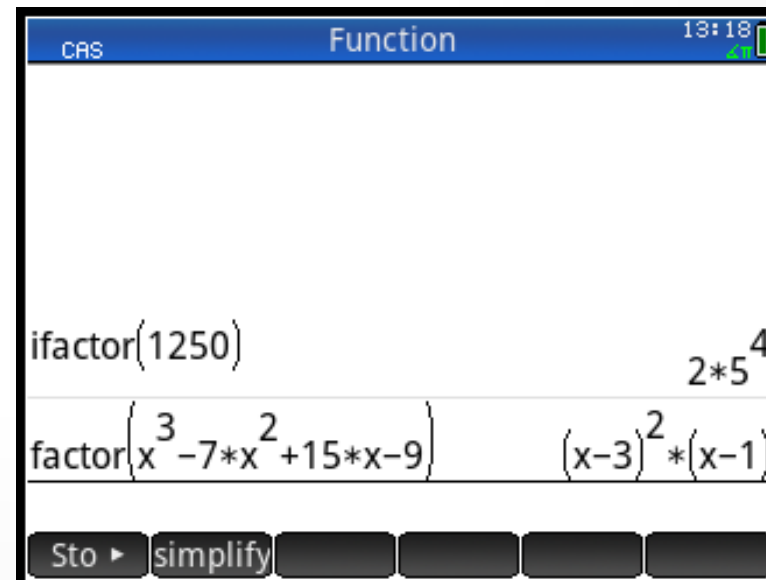


CAS Function 13:14

$\sqrt{44}$	$2\sqrt{11}$
$\text{ASIN}\left(\frac{1}{2}\right)$	$\frac{\pi}{6}$
$\frac{7}{16} + \frac{12}{8} - \frac{5}{12}$	$\frac{73}{48}$

Sto ► simplify

Factorization:
Integers and
Polynomials



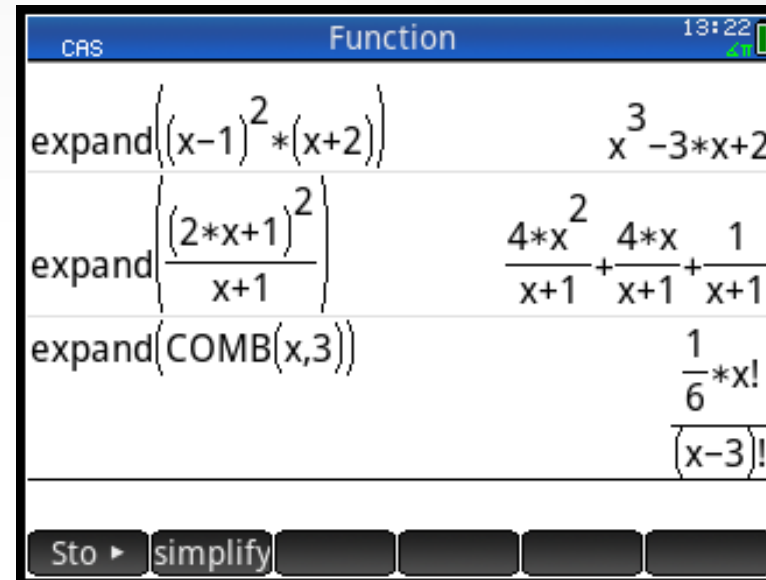
CAS Function 13:18

$\text{ifactor}(1250)$	$2 \cdot 5^4$
$\text{factor}(x^3 - 7x^2 + 15x - 9)$	$(x-3)^2 \cdot (x-1)$

Sto ► simplify

Some Essential CAS Functions

Expanding
Polynomials and
Expression with
Combinations

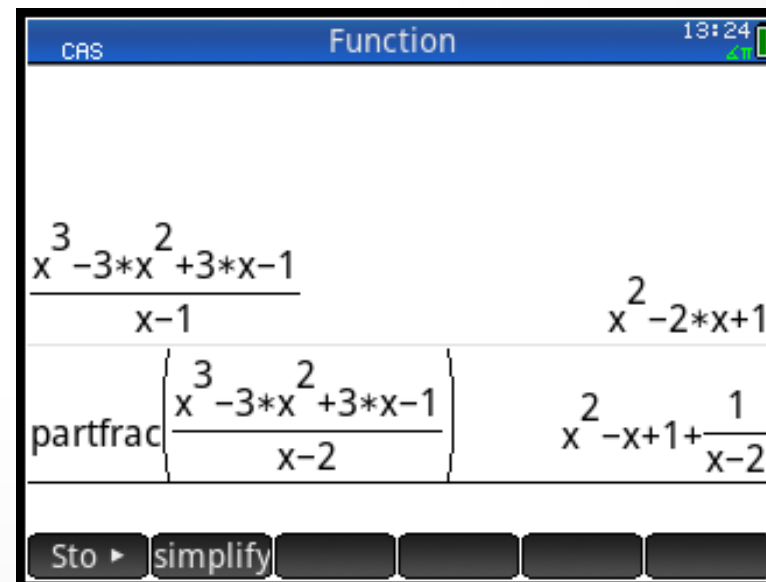


CAS Function 13:22

$\text{expand}\left\{\left(x-1\right)^2\cdot\left(x+2\right)\right\}$	x^3-3x+2
$\text{expand}\left\{\frac{\left(2x+1\right)^2}{x+1}\right\}$	$\frac{4x^2}{x+1}+\frac{4x}{x+1}+\frac{1}{x+1}$
$\text{expand}\left\{\text{COMB}\left(x,3\right)\right\}$	$\frac{1}{6}\cdot x!$ $\frac{1}{\left(x-3\right)!}$

Sto ► simplify

Synthetic Division



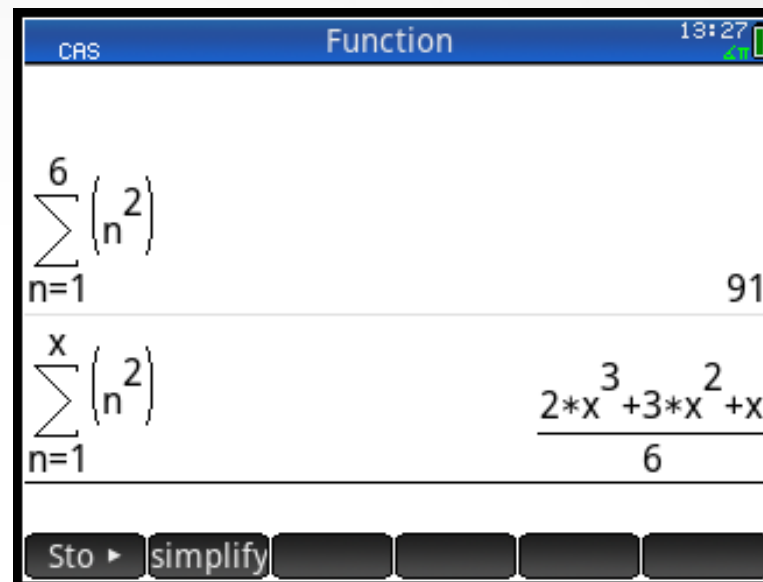
CAS Function 13:24

$\frac{x^3-3x^2+3x-1}{x-1}$	x^2-2x+1
$\text{partfrac}\left\{\frac{x^3-3x^2+3x-1}{x-2}\right\}$	$x^2-x+1+\frac{1}{x-2}$

Sto ► simplify

Some Essential CAS Functions

Sums: Numeric
and Symbolic



CAS Function 13:27

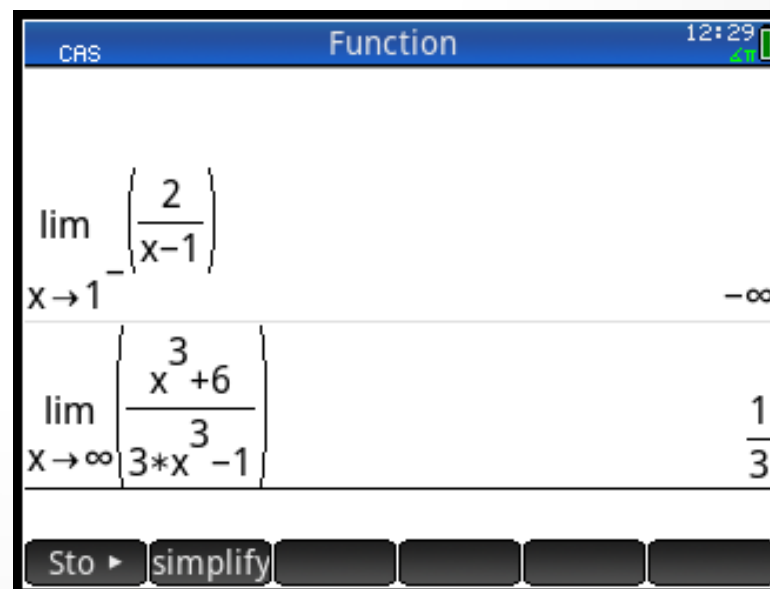
$\sum_{n=1}^6 (n^2)$ 91

$\sum_{n=1}^x (n^2)$ $\frac{2*x^3 + 3*x^2 + x}{6}$

Sto ► simplify

Detailed description: This screenshot shows a CAS interface with a blue header bar containing 'CAS', 'Function', and the time '13:27'. The main display area shows two sum calculations. The first is a numeric sum from n=1 to 6 of n squared, resulting in 91. The second is a symbolic sum from n=1 to x of n squared, resulting in the formula (2*x^3 + 3*x^2 + x) / 6. At the bottom, there is a row of buttons: 'Sto ►', 'simplify', and three empty buttons.

Limits



CAS Function 12:29

$\lim_{x \rightarrow 1^-} \left(\frac{2}{x-1} \right)$ $-\infty$

$\lim_{x \rightarrow \infty} \left(\frac{x^3 + 6}{3*x^3 - 1} \right)$ $\frac{1}{3}$

Sto ► simplify

Detailed description: This screenshot shows a CAS interface with a blue header bar containing 'CAS', 'Function', and the time '12:29'. The main display area shows two limit calculations. The first is the limit of 2/(x-1) as x approaches 1 from the left, resulting in -infinity. The second is the limit of (x^3 + 6) / (3*x^3 - 1) as x approaches infinity, resulting in 1/3. At the bottom, there is a row of buttons: 'Sto ►', 'simplify', and three empty buttons.

Some Essential CAS Functions

Derivatives: Numeric
and Symbolic

CAS Function 12:32

$$\left. \frac{\partial 4x^3 + x^2 - 3x + 1}{\partial x} \right|_{x=2} = 49$$
$$\frac{\partial 4x^3 + x^2 - 3x + 1}{\partial x} = 12x^2 + 2x - 3$$

Sto ► simplify

Integrals: Numeric
and Symbolic

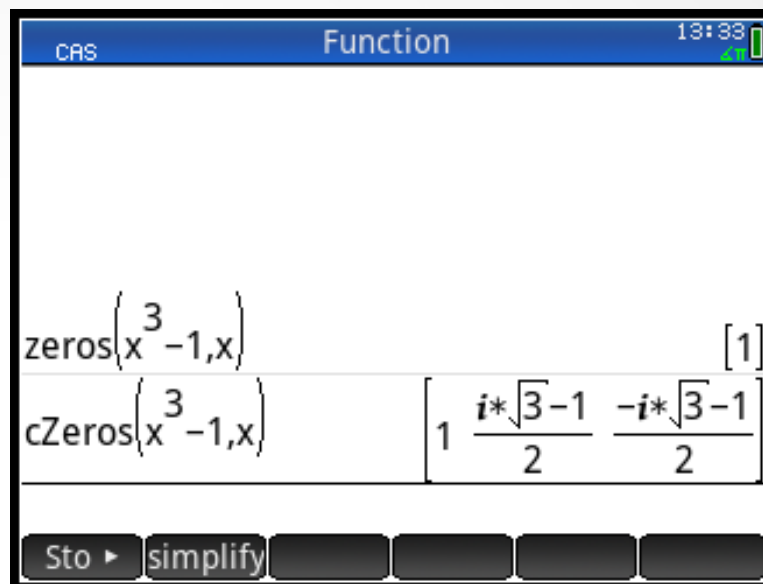
CAS Function 12:36

$$\int_0^t x \cdot \cos(x) dx = \frac{2 \cdot t \cdot \tan\left(\frac{t}{2}\right) - 2 \cdot \tan\left(\frac{t}{2}\right)^2}{\tan\left(\frac{t}{2}\right)^2 + 1}$$
$$\int_0^2 x \cdot \cos(x) dx = 0.402448017104$$

Sto ► simplify

Some Essential CAS Functions

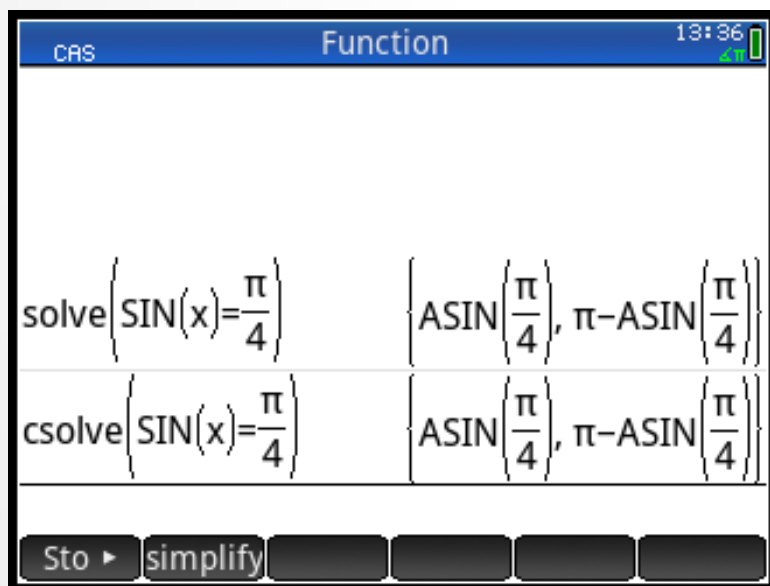
Real and Complex Numbered Solutions



CAS Function 13:33

$$\text{zeros}(x^3 - 1, x) \quad [1]$$
$$\text{cZeros}(x^3 - 1, x) \quad \left\{ 1, \frac{i\sqrt{3}-1}{2}, \frac{-i\sqrt{3}-1}{2} \right\}$$

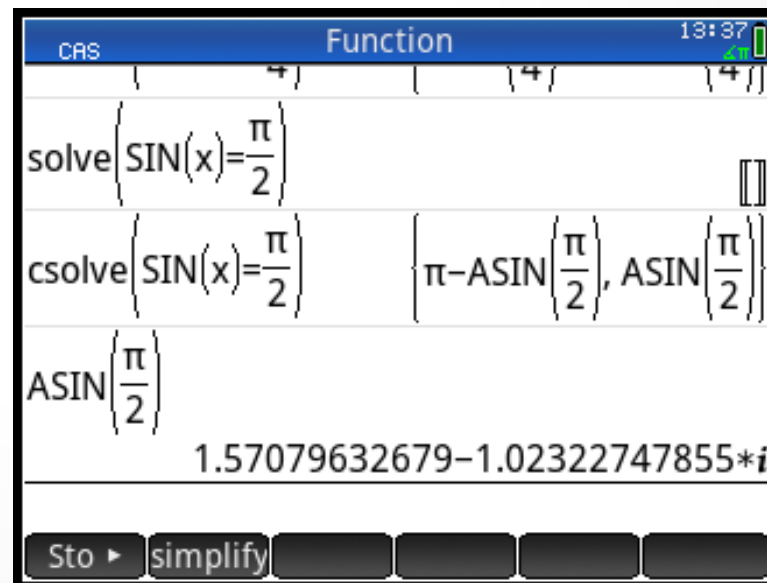
Sto ► simplify



CAS Function 13:36

$$\text{solve}\left(\text{SIN}(x) = \frac{\pi}{4}\right) \quad \left\{ \text{ASIN}\left(\frac{\pi}{4}\right), \pi - \text{ASIN}\left(\frac{\pi}{4}\right) \right\}$$
$$\text{csolve}\left(\text{SIN}(x) = \frac{\pi}{4}\right) \quad \left\{ \text{ASIN}\left(\frac{\pi}{4}\right), \pi - \text{ASIN}\left(\frac{\pi}{4}\right) \right\}$$

Sto ► simplify



CAS Function 13:37

$$\text{solve}\left(\text{SIN}(x) = \frac{\pi}{2}\right) \quad \left\{ \right\}$$
$$\text{csolve}\left(\text{SIN}(x) = \frac{\pi}{2}\right) \quad \left\{ \pi - \text{ASIN}\left(\frac{\pi}{2}\right), \text{ASIN}\left(\frac{\pi}{2}\right) \right\}$$
$$\text{ASIN}\left(\frac{\pi}{2}\right)$$
$$1.57079632679 - 1.02322747855 * i$$

Sto ► simplify

Challenges of Using CAS

- User is dependent on how complete and complex the CAS system is
- Bugs in the system
- Learning curve: can the user create programs with CAS functions
- Are CAS calculators allowed in the classroom?
- How to teach mathematics with CAS

Programming with CAS

- Your mileage may vary:
 - Limited and complex at best: Casio fx-Algebra series
 - High Learning Curve: HP Prime
 - Better integration with calculator features: TI-89/92 series (TI-Basic), HP 28/48/49/50g series (RPL)
- Programs allow automation with both numeric and symbolic inputs

CAS Program Example: HP Prime

```
castest03 12:57
#cas
castest03(f,a):=
BEGIN
// type f(x), value of x (a)
subst(f,x=a)
END;
#end
```

Cmds Tmplt Check

```
CAS Function 13:00
castest03(2*x^2+1,8) 129
castest03(2*x^2+1,(a-b)^2)
2*a^4-8*a^3*b+12*a^2*b^2-8*a*b^3+2*b^4+1
castest03(2*x^2+1,x^3/2) 2*x^3+1
```

Sto ► simplify

Educational and CAS

- Balance between use of CAS and learning how to solve problems manually
- How can CAS be used to effectively teach mathematics
- The use of CAS in presentations.

Educational and CAS

Explore Patterns

CAS Function 13:26

$x^2 * x^3$	x^5
$x^2 * x^4$	x^6
$x^2 * x^5$	x^7
$x^2 * x^6$	x^8

Sto ► simplify

Manual Step-by-Step Solving

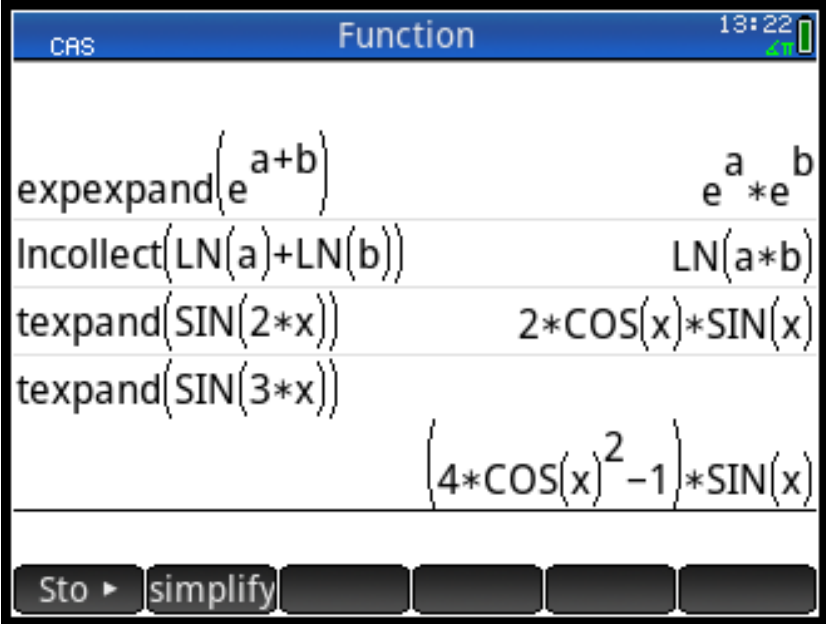
CAS Function 13:28

$8*x^2 - 66 = 0$	$8*x^2 - 66 = 0$
Ans+66	$8*x^2 = 66$
$\frac{\text{Ans}}{8}$	$x^2 = \frac{33}{4}$
$\sqrt{\text{Ans}}$	$ x = \frac{\sqrt{33}}{2}$

Sto ► simplify

Educational and CAS

Verify Trigonometric and Logarithmic Identities



The screenshot shows a CAS interface with a blue header bar containing 'CAS' on the left, 'Function' in the center, and '13:22' on the right. The main area displays four rows of mathematical identities, each with a function call on the left and its result on the right. The bottom of the screen features a row of buttons: 'Sto ►', 'simplify', and three empty buttons.

Function	Result
$\text{expexpand}(e^{a+b})$	$e^a * e^b$
$\text{Incollect}(\text{LN}(a)+\text{LN}(b))$	$\text{LN}(a*b)$
$\text{texpand}(\text{SIN}(2*x))$	$2*\text{COS}(x)*\text{SIN}(x)$
$\text{texpand}(\text{SIN}(3*x))$	$(4*\text{COS}(x)^2 - 1)*\text{SIN}(x)$

Challenges - Education

- Students may develop an over reliance on CAS
 - Calculators with Test Mode features, so teachers may turn off the calculator's CAS abilities
 - Students may not have the skills developed
- Learning curve of CAS commands
- Depending on settings, simplification skips middle steps
- Depending on the system used, range of problems may be limited

Future of CAS (in my opinion)

- Eventually every student will have a calculator with CAS capabilities
- Students will still need to know algebra, but with a working knowledge rather than hours-long practice
- CAS capabilities will enhance solving systems even in non-CAS calculators
 - Some calculator apps have a dedicated equation library, like the HP 48G/49/50g series

Thank You!
Questions?

